Problem ♡–8

Due in DSC 235 by 12 noon, Friday, November 03, 2017

Problem A: *Assume that a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous at an $x_0 \in \mathbb{R}$ and satisfies Cauchy functional equation:*

$$f(x+y) = f(x) + f(y) \qquad \text{for all } x, y \in \mathbb{R}.$$

Prove that $f(x) = f(1) \cdot x$ for all $x \in \mathbb{R}$.

Problem B: *Find all continuous functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ satisfying the Jensen equation*

$$f\Big(\frac{x+y}{2}\Big) = \frac{f(x)+f(y)}{2} \qquad \text{for all } x, y \in \mathbb{R}.$$

Problem C: *Prove that there is a function $f : \mathbb{R} \longrightarrow \mathbb{Q}$ satisfying the following three conditions:*

(1) *$f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$,*
(2) *$f(x) = x$ for all $x \in \mathbb{Q}$, and*
(3) *f is not continuous at any point on $\mathbb{R}$.*

RULES:

- The competition is open to all *undergraduate* UNO students.
- Please submit your solutions to Andrzej Roslanowski in DSC 235 or to his mailbox. (Needless to say, they should be be written clearly and legibly.)
- The winners will be determined each semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

PRIZES:

- Winners will received books published by the American Mathematical Society. The titles actually awarded will be selected in cooperation with the awardees.
- In Summer 2018, there is a research opportunity possibly that could lead to an Erdős Number (3 or possibly 2). Strong performance in POW is one of the crucial prerequisites.